

AUSTRALIAN MATHEMATICAL OLYMPIAD COMMITTEE

2013 IMO Team Training

Exam T17

- Each question is worth 7 points.
- Time allowed is $4\frac{1}{2}$ hours.
- No books, notes or calculators permitted.
- Any questions must be submitted in writing within the first half hour of the exam.

The 2013 Mathematical Ashes: AUS v UNK

1. Several positive integers are written in a row. Iteratively, Alice chooses two adjacent numbers x and y such that $x > y$ and x is to the left of y , and replaces the pair (x, y) by either $(y + 1, x)$ or $(x - 1, x)$.

Prove that Alice can perform only finitely many such iterations.

2. Determine all integers $m \geq 2$ such that every integer n with $\frac{m}{3} \leq n \leq \frac{m}{2}$ divides the binomial coefficient $\binom{n}{m-2n}$.
3. Let ABC be a triangle with circumcircle ω and let ℓ be a line which does not intersect ω . Let P be the foot of the perpendicular from the centre of ω to ℓ . The side-lines BC , CA and AB intersect ℓ , respectively, at the points X , Y and Z different from P .

Prove that the circumcircles of triangles AXP , BYP and CZP have a common point different from P or are mutually tangent at P .