Wednesday, July 5, 2023

Problem 1. Find all positive integers $n > 2$ such that

$$n! \;\Big|\; \prod_{\substack{p<q\le n, \\ p,q \text{ primes}}} (p+q).$$

Problem 2. In a particular form of solitaire, we start with n piles of pebbles (for some positive integer n), each initially containing a single pebble. A move consists of the following three operations: choosing two piles, taking an equal number of pebbles from each pile, and forming a new pile out of these pebbles.

For each positive integer n, find the smallest number of non-empty piles that one can obtain by performing a finite sequence of moves of this form.

Problem 3. Let $AA'BCC'B'$ be a convex cyclic hexagon such that AC is tangent to the incircle of triangle $A'B'C'$, and $A'C'$ is tangent to the incircle of triangle ABC. Let the lines AB and $A'B'$ meet at X, and let the lines BC and $B'C'$ meet at Y.

Prove that if $XBYB'$ is a convex quadrilateral, then it has an incircle.

Language: English
Questions must not be published

Time: 4 hours and 30 minutes
Each problem is worth 7 points