Saturday, July 12, 2025

Problem 1. Let n be a positive integer. A class of n students run n races, in each of which they are ranked with no draws. A student is eligible for a rating (a, b) for positive integers a and b if they come in the top b places in at least a of the races. Their final score is the maximum possible value of $a - b$ across all ratings for which they are eligible.

Find the maximum possible sum of all the scores of the n students.

Problem 2. Let $ABCD$ be a quadrilateral with AB parallel to CD and $AB < CD$. Lines AD and BC intersect at a point P. Point $X \neq C$ lies on the circumcircle of $\triangle ABC$ such that $PC = PX$. Point $Y \neq D$ lies on the circumcircle of $\triangle ABD$ such that $PD = PY$. Lines AX and BY intersect at point Q.

Prove that PQ is parallel to AB.

Problem 3. Given a positive integer m, we say that a polynomial P with integer coefficients is *m-ice* if there exists a polynomial Q of degree 2 with integer coefficients such that $Q(k)(P(k) + Q(k))$ is never divisible by m for any integer k.

Determine all integers n such that every polynomial with integer coefficients is an n-ice polynomial.