Saturday, July 11, 2026

Problem 1. Let ABC be an acute-angled triangle. Let I_B and I_C be the excentres of triangle ABC opposite B and C, respectively. Let E be a point on line BA such that $\angle EI_BB = 90°$. Let F be a point on line CA such that $\angle CI_CF = 90°$. Let the circle with centre I_B and radius I_BE intersect the segment I_BB at X. Let the circle with centre I_C and radius I_CF intersect the segment I_CC at Y.

Prove that BY is perpendicular to CX.

Problem 2. There are 2025 white balls and 2025 black balls arranged in a row. A *balanced segment* is a contiguous nonempty segment of balls that contains the same number of white balls as black balls. A *balanced removal* is the operation of selecting a balanced segment S, removing all the balls in S, and then shifting all the balls that were to the right of S to the left so that the remaining balls form a contiguous row.

Determine the smallest positive integer N satisfying the following property: for every configuration of balls and for every integer k satisfying $0 \leq k \leq 2025$, there exists a sequence of at most N balanced removals after which there are exactly $2k$ remaining balls.

Problem 3. For positive integers n and k, let $f_k(n)$ denote the remainder when n is divided by $2^k - 1$. A positive integer n is *grouse* if

$$f_1(n) \leq f_2(n) \leq f_3(n) \leq f_4(n) \leq \cdots .$$

(a) Prove that there are infinitely many grouse numbers.

(b) Prove that there exists a positive integer $M > 1$ such that there are at most $M/2025^{2025}$ grouse numbers less than M.