1. Consider the pair of four-digit positive integers 
\( (M, N) = (3600, 2500) \).
Notice that \( M \) and \( N \) are both perfect squares, with equal
digits in two places, and differing digits in the remaining
two places. Moreover, when the digits differ, the digit in \( M \)
is exactly one greater than the corresponding digit in \( N \).
Find all pairs of four-digit positive integers \((M, N)\) with these
properties.

2. A function \( f \) is defined over the set of all positive integers and
satisfies
\[
f(1) = 1996 \quad \text{and} \quad f(1) + f(2) + \cdots + f(n) = n^2 f(n) \quad \text{for all } n > 1.
\]
Calculate the exact value of \( f(1996) \).

3. Let \( \triangle ABC \) be an acute-angled triangle, and let \( O \) be its
circumcentre. The circle through \( A, O \) and \( B \) is called \( S \).
The lines \( CA \) and \( CB \) meet the circle \( S \) again at
\( P \) and \( Q \) respectively. Prove that the lines
\( CO \) and \( PQ \) are
perpendicular.

4. For any real number \( x \), let \([x]\) denote the greatest integer
which is less than or equal to \( x \). Define
\[
q(n) = \left\lfloor \frac{n}{\sqrt{n}} \right\rfloor \quad \text{for} \quad n = 1, 2, 3, \ldots
\]
Determine all positive integers \( n \) for which \( q(n) > q(n + 1) \).

5. Let \( a, b \) and \( c \) be positive real numbers.
(i) Prove that \( 4(a^3 + b^3) \geq (a + b)^3 \).
(ii) Prove that \( 9(a^3 + b^3 + c^3) \geq (a + b + c)^3 \).