BRITISH MATHEMATICAL OLYMPIAD

Round 1 : Wednesday, 17 January 2001

Time allowed *Three and a half hours.*

Instructions

- *Full written solutions - not just answers - are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt. Do not hand in rough work.*
- *One* complete *solution will gain far more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all five problems.*
- *Each question carries 10 marks.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Start each question on a fresh sheet of paper. Write on one side of the paper only. On each sheet of working write the number of the question in the top* left *hand corner and your name, initials and school in the top* right *hand corner.*
- *Complete the cover sheet provided and attach it to the front of your script, followed by the questions 1,2,3,4,5 in order.*
- *Staple all the pages neatly together in the top* left *hand corner.*

Do not turn over until **told to do so.**

2001 British Mathematical Olympiad
Round 1

1. Find all two-digit integers N for which the sum of the digits of $10^N - N$ is divisible by 170.

2. Circle S lies inside circle T and touches it at A. From a point P (distinct from A) on T, chords PQ and PR of T are drawn touching S at X and Y respectively. Show that $\angle QAR = 2\angle XAY$.

3. A *tetromino* is a figure made up of four unit squares connected by common edges.
   (i) If we do not distinguish between the possible rotations of a tetromino within its plane, prove that there are seven distinct tetrominoes.
   (ii) Prove or disprove the statement: It is possible to pack all seven distinct tetrominoes into a 4×7 rectangle without overlapping.

4. Define the sequence (a_n) by
$$a_n = n + \{\sqrt{n}\},$$
where n is a positive integer and $\{x\}$ denotes the nearest integer to x, where halves are rounded up if necessary. Determine the smallest integer k for which the terms $a_k, a_{k+1}, \ldots, a_{k+2000}$ form a sequence of 2001 consecutive integers.

5. A triangle has sides of length a, b, c and its circumcircle has radius R. Prove that the triangle is right-angled if and only if $a^2 + b^2 + c^2 = 8R^2$.