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British Mathematical Olympiad

Round 1 : Wednesday, 1 December 2004

Time allowed *Three and a half hours.*

Instructions

- *Full written solutions - not just answers - are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt. Do not hand in rough work.*
- *One* complete *solution will gain far more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all five problems.*
- *Each question carries 10 marks.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Start each question on a fresh sheet of paper. Write on one side of the paper only. On each sheet of working write the number of the question in the top* left *hand corner and your name, initials and school in the top* right *hand corner.*
- *Complete the cover sheet provided and attach it to the front of your script, followed by the questions 1,2,3,4,5 in order.*
- *Staple all the pages neatly together in the top* left *hand corner.*

Do not turn over until **told to do so.**

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2004/5 British Mathematical Olympiad
Round 1

1. Each of Paul and Jenny has a whole number of pounds.
He says to her: "If you give me £3, I will have n times as much as you".
She says to him: "If you give me £n, I will have 3 times as much as you".
Given that all these statements are true and that n is a positive integer, what are the possible values for n?

2. Let ABC be an acute-angled triangle, and let D, E be the feet of the perpendiculars from A, B to BC, CA respectively. Let P be the point where the line AD meets the semicircle constructed outwardly on BC, and Q be the point where the line BE meets the semicircle constructed outwardly on AC. Prove that $CP = CQ$.

3. Determine the least natural number n for which the following result holds:
No matter how the elements of the set $\{1, 2, \ldots, n\}$ are coloured red or blue, there are integers x, y, z, w in the set (not necessarily distinct) of the same colour such that $x + y + z = w$.

4. Determine the least possible value of the largest term in an arithmetic progression of seven distinct primes.

5. Let S be a set of rational numbers with the following properties:
i) $\frac{1}{2} \in S$;
ii) If $x \in S$, then both $\frac{1}{x+1} \in S$ and $\frac{x}{x+1} \in S$.
Prove that S contains all rational numbers in the interval $0 < x < 1$.