United Kingdom Mathematics Trust

British Mathematical Olympiad

Round 1 : Friday, 30 November 2012

Time allowed $3\frac{1}{2}$ hours.

Instructions

- *Full written solutions – not just answers – are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.*
- *One* complete *solution will gain more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all the problems.*
- *Each question carries 10 marks. However, earlier questions tend to be easier. In general you are advised to concentrate on these problems first.*
- *The use of rulers, set squares and compasses is allowed, but calculators and protractors are forbidden.*
- *Start each question on a fresh sheet of paper. Write on one side of the paper only. On each sheet of working write the number of the question in the top* left *hand corner and your name, initials and school in the top* right *hand corner.*
- *Complete the cover sheet provided and attach it to the front of your script, followed by your solutions in question number order.*
- *Staple all the pages neatly together in the top* left *hand corner.*
- *To accommodate candidates sitting in other time-zones, please do not discuss the paper on the internet until 8am GMT on Saturday 1 December.*

Do not turn over until **told to do so.**

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2012/13 British Mathematical Olympiad

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1. Isaac places some counters onto the squares of an 8 by 8 chessboard so that there is at most one counter in each of the 64 squares. Determine, with justification, the maximum number that he can place without having five or more counters in the same row, or in the same column, or on either of the two long diagonals.

2. Two circles S and T touch at X. They have a common tangent which meets S at A and T at B. The points A and B are different. Let AP be a diameter of S. Prove that B, X and P lie on a straight line.

3. Find all real numbers x, y and z which satisfy the simultaneous equations $x^2 - 4y + 7 = 0$, $y^2 - 6z + 14 = 0$ and $z^2 - 2x - 7 = 0$.

4. Find all positive integers n such that $12n - 119$ and $75n - 539$ are both perfect squares.

5. A triangle has sides of length at most 2, 3 and 4 respectively. Determine, with proof, the maximum possible area of the triangle.

6. Let ABC be a triangle. Let S be the circle through B tangent to CA at A and let T be the circle through C tangent to AB at A. The circles S and T intersect at A and D. Let E be the point where the line AD meets the circle ABC. Prove that D is the midpoint of AE.