1. Calculate the value of
\[
\frac{2014^4 + 4 \times 2013^4}{2013^2 + 4027^2} - \frac{2012^4 + 4 \times 2013^4}{2013^2 + 4025^2}.
\]

2. In the acute-angled triangle $ABC$, the foot of the perpendicular from $B$ to $CA$ is $E$. Let $l$ be the tangent to the circle $ABC$ at $B$. The foot of the perpendicular from $C$ to $l$ is $F$. Prove that $EF$ is parallel to $AB$.

3. A number written in base 10 is a string of 32013 digit 3s. No other digit appears. Find the highest power of 3 which divides this number.

4. Isaac is planning a nine-day holiday. Every day he will go surfing, or water skiing, or he will rest. On any given day he does just one of these three things. He never does different water-sports on consecutive days. How many schedules are possible for the holiday?

5. Let $ABC$ be an equilateral triangle, and $P$ be a point inside this triangle. Let $D, E$ and $F$ be the feet of the perpendiculars from $P$ to the sides $BC, CA$ and $AB$ respectively. Prove that
   a) $AF + BD + CE = AE + BF + CD$ and
   b) $[APF] + [BPD] + [CPE] = [APE] + [BPF] + [CPD].$
   The area of triangle $XYZ$ is denoted $[XYZ]$.

6. The angles $A, B$ and $C$ of a triangle are measured in degrees, and the lengths of the opposite sides are $a, b$ and $c$ respectively. Prove that
\[
60 \leq \frac{aA + bB + cC}{a + b + c} < 90.
\]