United Kingdom
Mathematics Trust

British Mathematical Olympiad
Round 1

Thursday 26 November 2020

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Instructions

1. Time allowed: $2\frac{1}{2}$ hours.
2. Each question in Section A carries 5 marks. Each question in Section B carries 10 marks. Earlier questions tend to be easier; you are advised to concentrate on these problems first.
3. In Section A only answers are required.
4. Use the answer sheet provided for Section A.
5. In Section B full written solutions – not just answers – are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.
6. One complete solution will gain more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all the problems.
7. Write on one side of the paper only. Start each question in Section B on a fresh sheet of paper: *scans of your work will need to be uploaded question by question for marking.*
8. On each sheet of working for Section B, write the number of the question in the top left hand corner and your **Participant ID** and **UKMT centre number**. Do not write your name.
9. The use of rulers, set squares and compasses is allowed, but calculators and protractors are forbidden. *You are strongly encouraged to use geometrical instruments to construct large, accurate diagrams for Section B geometry problems.*
10. At the end of the paper, return to your Section A answer sheet and indicate which Section B questions you have attempted.
11. Please do not discuss the paper on the internet until 5pm GMT on Wednesday 2 December when the solutions video will be released at bmos.ukmt.org.uk
12. **Do not turn over until told to do so.**

Enquiries about the British Mathematical Olympiad should be sent to:

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Section A

The questions in Section A are worth a maximum of five points each.
Only answers are required.
Please use the answer sheet provided.

1. Alice and Bob take it in turns to write numbers on a blackboard. Alice starts by writing an integer a between -100 and 100 inclusive on the board. On each of Bob's turns he writes twice the number Alice wrote last. On each of Alice's subsequent turns she writes the number 45 less than the number Bob wrote last. At some point, the number a is written on the board for a second time. Find the possible values of a.

2. A triangle has side lengths a, a and b. It has perimeter P and area A. Given that b and P are integers, and that P is numerically equal to A^2, find all possible pairs (a, b).

3. A square piece of paper is folded in half along a line of symmetry. The resulting shape is then folded in half along a line of symmetry of the new shape. This process is repeated until n folds have been made, giving a sequence of $n+1$ shapes. If we do not distinguish between congruent shapes, find the number of possible sequences when:

(a) $n = 3$;

(b) $n = 6$;

(c) $n = 9$.

(When $n = 1$ there are two possible sequences.)

4. In the equation

$$\mathsf{A}^{\mathsf{AA}} + \mathsf{AA} = \mathsf{B{,}BBC{,}DED{,}BEE{,}BBB{,}BBE}$$

the letters A, B, C, D and E represent different base 10 digits (so the right hand side is a sixteen digit number and AA is a two digit number). Given that C = 9, find A, B, D and E.

Section B

The questions in Section B are worth a maximum of ten points each.
Full written solutions are required.
Please begin each question on a new sheet of paper.

5. Let points A, B and C lie on a circle Γ. Circle Δ is tangent to AC at A. It meets Γ again at D and the line AB again at P. The point A lies between points B and P. Prove that if $AD = DP$, then $BP = AC$.

6. Given that an integer n is the sum of two different powers of 2 and also the sum of two different Mersenne primes, prove that n is the sum of two different square numbers.

(A Mersenne prime *is a prime number which is one less than a power of two.)*

7. Evie and Odette are playing a game. Three pebbles are placed on the number line; one at -2020, one at 2020, and one at n, where n is an integer between -2020 and 2020. They take it in turns moving either the leftmost or the rightmost pebble to an integer between the other two pebbles. The game ends when the pebbles occupy three consecutive integers.

Odette wins if their sum is odd; Evie wins if their sum is even. For how many values of n can Evie guarantee victory if:

(a) Odette goes first;

(b) Evie goes first?

