United Kingdom
Mathematics Trust

British Mathematical Olympiad

Round 1

Thursday 2 December 2021

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Instructions

1. Time allowed: $3\frac{1}{2}$ hours.
2. Full written solutions – not just answers – are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.
3. One complete solution will gain more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all the problems.
4. Each question carries 10 marks. However, earlier questions tend to be easier. In general you are advised to concentrate on these problems first.
5. The use of rulers, set squares and compasses is allowed, but calculators and protractors are forbidden. *You are strongly encouraged to use geometrical instruments to construct large, accurate diagrams for geometry problems.*
6. Start each question on an official answer sheet on which there is a QR code.
7. If you use additional sheets of (plain or lined) paper for a question, please write the following in the top left-hand corner of each sheet. (i) The question number. (ii) The page number for that question. (iii) The digits following the ':' from the question's answer sheet QR code.
8. Write on one side of the paper only. Make sure your writing and diagrams are clear and not too faint. (*Your work will be scanned for marking.*)
9. Arrange your answer sheets in question order before they are collected. If you are not submitting work for a particular problem, please remove the associated answer sheet.
10. To accommodate candidates sitting in other time zones, please do not discuss the paper on the internet until 8am on Saturday 4 December UK time. when the solutions video will be released at https://bmos.ukmt.org.uk
11. **Do not turn over until told to do so.**

Enquiries about the British Mathematical Olympiad should be sent to:

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1. Find three even numbers less than 400, each of which can be expressed as a sum of consecutive positive odd numbers in at least six different ways.

(Two expressions are considered to be different if they contain different numbers. The order of the numbers forming a sum is irrelevant.)

2. One day Arun and Disha played several games of table tennis. At five points during the day, Arun calculated the percentage of the games played so far that he had won. The results of these calculations were exactly 30%, exactly 40%, exactly 50%, exactly 60% and exactly 70% in some order. What is the smallest possible number of games they played?

3. For each integer $0 \le n \le 11$, Eliza has exactly three identical pieces of gold that weigh 2^n grams. In how many different ways can she form a pile of gold weighing 2021 grams?

(*Two piles are different if they contain different numbers of gold pieces of some weight. The arrangement of the pieces in the piles is irrelevant.*)

4. Two circles Γ_1 and Γ_2 have centres O_1 and O_2 respectively. They pass through each other's centres and intersect at A and B. The point C lies on the minor arc BO_2 of Γ_1. The points D and E lie on the line O_2C such that $\angle AO_1D = \angle DO_1C$ and $\angle CO_1E = \angle EO_1B$. Prove that triangle DO_1E is equilateral.

(A minor arc of a circle is the shorter of the two arcs with given endpoints.)

5. An N-set is a set of different positive integers including a given positive integer N. Let $m(N)$ be the smallest possible mean of any N-set. For how many values of N less than 2021 is $m(N)$ an integer?

6. Marvin has been tasked with writing down every list of integers with the following properties:

(i) The list contains 71 terms.

(ii) The first term is 1.

(iii) Every term after the first is equal to either the previous term, or the sum of all previous terms.

When Marvin is finished, how many of the lists will have a sum equal to 999,999?

