UK Maths Trust

British Mathematical Olympiad

Round 1

Wednesday 19 November 2025

supported by Jane Street Overleaf

Instructions

1. Time allowed: $\mathbf{3\frac{1}{2}}$ **hours**.
2. **Full written solutions – not just answers – are required**, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.
3. **One complete solution will gain more credit than several unfinished attempts**. It is more important to complete a small number of questions than to try all the problems.
4. **Each question carries 10 marks**. However, earlier questions tend to be easier. In general you are advised to concentrate on these problems first.
5. The use of rulers, set squares and compasses is allowed, but **calculators and protractors are forbidden**. You are strongly encouraged to use geometrical instruments to construct large, accurate diagrams for geometry problems.
6. Start each question on an official answer sheet on which there is a **QR code**.
7. If you use additional sheets of (plain or lined) paper for a question, please write the following in the top left-hand corner of each sheet. (i) The question number. (ii) The page number for that question. (iii) The digits following the ':' from the question's answer sheet QR code. **Please do not write your name or initials on additional sheets**.
8. **Write on one side of the paper only**. Make sure your writing and diagrams are clear and not too faint. (*Your work will be scanned for marking*.)
9. **Arrange your answer sheets in question order before they are collected**. If you are not submitting work for a particular problem, please remove the associated answer sheet.
10. To accommodate candidates sitting in other time zones, please do not discuss or share the paper on the internet (including in email) until **8am GMT on Friday 21st November** when the solutions video will be released at https://bmos.ukmt.org.uk. Candidates in time zones more than 5 hours ahead of GMT must sit the paper on Thursday 20th November (as defined locally).
11. **Do not turn over until told to do so**.

Enquiries about the British Mathematical Olympiad should be sent to:

challenges@ukmt.org.uk

www.ukmt.org.uk

1. A *ramp* is a sequence of three different positive integers a, b, c such that a is a factor of b and b is a factor of c. For every prime number p and every positive integer n, determine with proof whether p^n can be expressed as the sum of a ramp.

2. Find all triples (x, y, z) of real numbers satisfying the equations

$$x^2 + 2yz = 4, \quad y^2 + 2zx = 4, \quad z^2 + 2xy = 1.$$

3. An 11×11 square grid is dissected into 11 pieces as follows. At each step, remove the top row and the leftmost column of the remaining grid as a single piece. Thus, the pieces obtained are L-shaped with sizes $21, 19, \ldots, 3$, together with a single square.

The pieces are rearranged to reassemble an 11×11 grid, with rotations allowed (but the pieces may not be turned over) so that the single square occupies the cell in the third row and third column (counting from the top left).

How many ways can this be done?

4. Let ABC be an acute-angled triangle with $AB > AC$. Let M be the midpoint of BC. The circle passing through M that is tangent to AB at B and the circle passing through M that is tangent to AC at C intersect again at D.

Prove that $MA \times MD = MB \times MC$.

5. George defines a sequence of positive integers $t_1, t_2, t_3, \ldots$ as follows.

He first sets $t_1 = 1$. Then, for $n \geq 1$:

- If t_n is even, then $t_{n+1} = t_n/2$.
- If t_n is odd and greater than 1, then t_{n+1} is $t_n/3$ rounded to the nearest integer.
- If $t_n = 1$, then $t_{n+1} = 2025k$ where k is the number of terms equal to 1 amongst $t_1, t_2, \ldots, t_n$.

Does this sequence contain every positive integer?

6. There are 1000 lily pads on a pond arranged in a circle and labelled $1, 2, \ldots, 1000$ in order. (The first four lily pads may also be described using the labels 1001, 1002, 1003 and 1004, respectively.) The first n lily pads are each occupied by a frog with the remaining lily pads not occupied.

Each minute, exactly one of the frogs makes a move. Suppose the frog is on lily pad k. That frog may either:

(i) Swim to lily pad $k + 4$ or $k - 4$, provided that it is not occupied; or

(ii) Jump to lily pad $k + 3$ or $k - 3$ provided that this lily pad is not occupied and the two lily pads jumped over are both occupied. When this happens, the two frogs that were jumped over dive into the pond and don't participate in any further moves.

For which values of n is it possible, by a sequence of moves, to end with exactly one frog remaining on the lily pads?

