UK Maths Trust

British Mathematical Olympiad

Round 1

Wednesday 18 November 2026

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Instructions

1. Time allowed: $\mathbf{3\frac{1}{2}}$ **hours**.
2. **Full written solutions – not just answers – are required**, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.
3. **One complete solution will gain more credit than several unfinished attempts**. It is more important to complete a small number of questions than to try all the problems.
4. **Each question carries 10 marks**. However, earlier questions tend to be easier. In general you are advised to concentrate on these problems first.
5. The use of rulers, set squares and compasses is allowed, but **calculators and protractors are forbidden**. You are strongly encouraged to use geometrical instruments to construct large, accurate diagrams for geometry problems.
6. Start each question on an official answer sheet on which there is a **QR code**.
7. If you use additional sheets of (plain or lined) paper for a question, please write the following in the top left-hand corner of each sheet. (i) The question number. (ii) The page number for that question. (iii) The digits following the ':' from the question's answer sheet QR code. **Please do not write your name or initials on additional sheets**.
8. **Write on one side of the paper only**. Make sure your writing and diagrams are clear and not too faint. (*Your work will be scanned for marking*.)
9. **Arrange your answer sheets in question order before they are collected**. If you are not submitting work for a particular problem, please remove the associated answer sheet.
10. To accommodate candidates sitting in other time zones, please do not discuss the paper on the internet until **8am GMT on Friday 20th November** when the solutions video will be released at https://bmos.ukmt.org.uk
11. **Do not turn over until told to do so**.

Enquiries about the British Mathematical Olympiad should be sent to:

challenges@ukmt.org.uk

www.ukmt.org.uk

1. Each of the integers $1, 2, 3, \ldots, 26$ is coloured either red or blue, and at least two of them are red. The colouring is called *snug* if the difference between the largest red number and the smallest red number is equal to the number of red numbers.

Determine, with proof, the number of snug colourings.

2. Let m and n be positive integers with $m < n$. Henry has a secret list of n non-zero real numbers $a_1, a_2, \ldots, a_n$, each of which may be positive or negative. George knows m and n, but not the list of numbers. George may choose any m of the n positions in the list, and Henry will then tell him the product of the m numbers in those positions. George may do this as many times as he likes.

Determine, with proof, all pairs (m, n) for which George can be certain to work out which number is in each position of the list, whatever numbers Henry has chosen.

3. Two circles Γ and Λ lie inside a circle Ω and do not meet each other. The circle Γ touches Ω at A, and the circle Λ touches Ω at B. A line touches Γ at D and touches Λ at C, and the circles Γ and Λ lie on the same side of this line. The centre of Ω lies strictly inside quadrilateral $ABCD$.

Prove that $ABCD$ is a cyclic quadrilateral.

4. A sequence $x_1, x_2, x_3, \ldots$ of real numbers is called *Fibonacci-like* if $x_{n+2} = x_{n+1} + x_n$ for every positive integer n.

Katy chooses three Fibonacci-like sequences $a_1, a_2, \ldots$ and $b_1, b_2, \ldots$ and $c_1, c_2, \ldots$ and, for each positive integer n, compares the three numbers a_n, b_n and c_n. She then writes down a three-letter string made of the letters a, b and c in increasing order of these three numbers. For example, if $a_1 < b_1 < c_1$ then she writes abc for $n = 1$, and if $a_2 < c_2 < b_2$ then she writes acb for $n = 2$. If two or more of a_n, b_n and c_n are equal, then she writes nothing for that value of n.

Determine, with proof, the largest possible number of different strings that Katy could write down.

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5. A tower has infinitely many floors, numbered $1, 2, 3, \ldots$ from the bottom. Sam wants to get down to floor 1. When he is on floor $n \geq 2$, a move consists of choosing a positive factor d of n with $d < n$ and going down d floors, to floor $n - d$.

For each integer $n \geq 2$, let $t(n)$ be the smallest number of moves in which Sam can get from floor n to floor 1. We say that floor n is *special* if $t(m) \geq t(n)$ for every integer $m > n$. For example, floor 3 is special, but floor 7 is not, since $t(7) = 4$ and $t(8) = 3$.

Determine all integers $n \geq 2$ for which floor n is special.

6. Four different points are marked in the plane. An *operation* consists of choosing one of the four points and a line passing through two of the other three points (the two points may be in the same position), and replacing the chosen point by its reflection in that line. We say that a point is *close* to a line if the perpendicular distance from the point to the line is less than $\frac{1}{2026}$.

Prove that it is always possible to carry out a finite sequence of operations after which at least one of the four points is close to a line passing through two of the others.

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