United Kingdom Mathematics Trust

British Mathematical Olympiad

Round 2 : Thursday, 31 January 2008

Time allowed *Three and a half hours.*

Each question is worth 10 marks.

Instructions

- *Full written solutions - not just answers - are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt.*

  Rough work should *be handed in, but should be clearly marked.*
- *One or two* complete *solutions will gain far more credit than partial attempts at all four problems.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Staple all the pages neatly together in the top* left *hand corner, with questions 1,2,3,4 in order, and the cover sheet at the front.*

In early March, twenty students will be invited to attend the training session to be held at Trinity College, Cambridge (3-7 April). At the training session, students sit a pair of IMO-style papers and 8 students will be selected for further training. Those selected will be expected to participate in correspondence work and to attend further training. The UK Team of 6 for this summer's International Mathematical Olympiad (to be held in Madrid, Spain 14-22 July) will then be chosen.

Do not turn over until **told to do so.**

United Kingdom Mathematics Trust

2007/8 British Mathematical Olympiad

Round 2

1. Find the minimum value of $x^2 + y^2 + z^2$ where x, y, z are real numbers such that $x^3 + y^3 + z^3 - 3xyz = 1$.

2. Let triangle ABC have incentre I and circumcentre O. Suppose that $\angle AIO = 90°$ and $\angle CIO = 45°$. Find the ratio $AB : BC : CA$.

3. Adrian has drawn a circle in the xy-plane whose radius is a positive integer at most 2008. The origin lies somewhere inside the circle. You are allowed to ask him questions of the form "Is the point (x, y) inside your circle?" After each question he will answer truthfully "yes" or "no". Show that it is always possible to deduce the radius of the circle after at most sixty questions. [*Note: Any point which lies exactly on the circle may be considered to lie inside the circle.*]

4. Prove that there are infinitely many pairs of distinct positive integers x, y such that $x^2 + y^3$ is divisible by $x^3 + y^2$.