



United Kingdom
Mathematics Trust

BRITISH MATHEMATICAL OLYMPIAD

ROUND 2

Thursday 28 January 2021

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INSTRUCTIONS

1. Time allowed: $3\frac{1}{2}$ hours. Each question is worth 10 marks.
2. Full written solutions – not just answers – are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt.
3. One or two *complete* solutions will gain far more credit than partial attempts at all four problems.
4. Write on one side of the paper only and start each question on a fresh sheet.
5. You should write in blue or black ink, but may use pencil and other colours for diagrams.
6. You may hand in rough work for each question where it contains calculations, examples or ideas not present in your final attempt; write ‘ROUGH’ at the top of each page of rough work.
7. The use of rulers and compasses is allowed, but calculators and protractors are forbidden.
8. Write your candidate number and UKMT centre number neatly in the top left corner of each page and arrange them so that your teacher can easily upload them to the marking platform.
9. To accommodate candidates sitting in other time zones, please do not discuss any aspect of the paper on the internet until 9am GMT on Friday 29 January. Candidates sitting the paper in time zones more than 3 hours ahead of GMT must sit the paper on the morning of Friday 29 January (as defined locally).
10. In early March, top-scoring students eligible to represent the UK at the International Mathematical Olympiad will be invited to attend a week of sessions, which will be held in an online format during the Easter holidays, comprising training for olympiads and general mathematical interest. Tests to select the UK team of six for this year’s IMO (to be hosted by Russia, possibly in a virtual format, 14–24 July 2021) will take place after the training week.
11. **Do not turn over until told to do so.**

Enquiries about the British Mathematical Olympiad should be sent to:

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1. A positive integer n is called *good* if there is a set of divisors of n whose members sum to n and include 1. Prove that every positive integer has a multiple which is good.

2. Eliza has a large collection of $a \times a$ and $b \times b$ tiles where a and b are positive integers. She arranges some of these tiles, without overlaps, to form a square of side length n . Prove that she can cover another square of side length n using only one of her two types of tile.

3. Let ABC be a triangle with $AB > AC$. Its circumcircle is Γ and its incentre is I . Let D be the contact point of the incircle of ABC with BC .
Let K be the point on Γ such that $\angle AKI$ is a right angle.
Prove that AI and KD meet on Γ .

4. Matthew writes down a sequence a_1, a_2, a_3, \dots of positive integers. Each a_n is the smallest positive integer, different from all previous terms in the sequence, such that the mean of the terms a_1, a_2, \dots, a_n is an integer. Prove that the sequence defined by $a_i - i$ for $i = 1, 2, 3, \dots$ contains every integer exactly once.