UK Maths Trust

British Mathematical Olympiad

Round 2

Wednesday 21 January 2026

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Instructions

1. Do not turn over until the invigilator tells you to do so.
2. Time allowed: **$3\frac{1}{2}$ hours**.
3. **Full written solutions – not just answers – are required**, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt.
4. **You may hand in rough work** where it contains calculations, examples or ideas not present in your final attempt; write 'ROUGH' at the top of each page of rough work.
5. **Each question carries 10 marks**. One or two complete solutions will gain far more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all the problems.
6. The use of rulers and compasses is allowed, but **calculators and protractors are forbidden**. You are strongly encouraged to use geometrical instruments to construct large, accurate diagrams for geometry problems.
7. Start each question on an official answer sheet on which there is a **QR code**.
8. If you use additional sheets of (plain or lined) paper for a question, please write the following in the top left-hand corner of each sheet. (i) The question number. (ii) The page number for that question. (iii) The digits following the ':' from the question's answer sheet QR code. **Please do not write your name or initials on any additional sheets**.
9. **Write on one side of the paper only**. Make sure your writing and diagrams are not too faint. (Your work will be scanned for marking.)
10. **Arrange your answer sheets, including rough work, in question order before they are collected**. If you are not submitting work for a particular problem, remove the associated answer sheet.
11. To accommodate candidates in other time zones, please do not discuss any aspect of the paper on the internet until **8am GMT on Friday 23rd January** when the solutions video will be released as https://bmos.ukmt.org.uk. Candidates in time zones more than 2 hours ahead of GMT must sit the paper on Thursday 22nd January 2026 (as defined locally).
12. Around 24 high-scoring students eligible to represent the UK at the International Mathematical Olympiad will be invited to a training session held in Cambridge around the Easter holidays.

Enquiries about the British Mathematical Olympiad should be sent to: *challenges@ukmt.org.uk*

1. For any two positive integers m and n, we define $l(m,n)$ as their least common multiple and $h(m,n)$ as their highest common factor. Given a prime $p > 3$, let k denote the number of ordered pairs of positive integers (m,n) satisfying the equation

$$l(m,n) + h(m,n) = p^4.$$

Determine the smallest possible value of k across all choices of the prime $p > 3$.

2. The convex quadrilateral $ABCD$ has sides AD and BC not parallel. The diagonals AC and BD intersect at X. The perpendicular bisectors of sides AD and BC meet at Y. Suppose that Y lies strictly inside triangle XCD.

Prove that $AC = BD$ if and only if XY bisects $\angle DXC$.

3. Each cell of a 30×30 grid contains one of the numbers -1, 0 or 1 with each of these three numbers appearing exactly 300 times.

Is it possible that the 60 row and column sums are all different?

4. Let N be a positive integer and let $(k_n)_{n\geq 1}$ be a sequence of positive integers with all terms at most N. Annabel begins by choosing integers $x_1, x_2, \ldots, x_N$. She then extends this to an infinite sequence $(x_n)_{n\geq 1}$ of integers by defining

$$x_n = \sum_{i=n-k_n}^{n-1} x_i$$

for each $n > N$.

Show that there are either finitely many strictly positive terms or finitely many strictly negative terms in the infinite sequence (x_n).

