BMO Marking

Each question on the British Mathematical Olympiad is marked out of a total of 10 marks. The style of marking is rather different to that which most students experience at school, and candidates are often surprised to find that they score fewer than 10 marks on questions which they think they have solved completely. In order to illustrate what the BMO examiners are looking for, and where candidates may lose marks, we give **simplified** markschemes for two questions from the 2002–3 first round paper.

1. **Given that**

$$34! = 295,232,799,cd9,604,140,847,618,609,643,5ab,000,000$$

determine the digits a, b, c, d.

Very few marks are awarded at the BMO for correct *answers* alone. In order for a candidate to receive substantial credit on any question, she must carefully explain all her reasoning. The correct answer to this question is $(a, b, c, d) = (2, 0, 0, 3)$. However, a candidate who merely asserted this without justification would receive no marks at all!

Almost every approach to this question begins by first calculating the values of a and b, and then moves on to calculate c and d. In order to prove that $b = 0$, it is sufficient to note that 10^7 divides 34!, so that 34! ends in at least 7 zeros.

$b = 0$ **with reasoning [2]**

The neatest way to show that $a = 2$ is to consider the value of $34!/10^7$ modulo 8. A more cumbersome method is to work out the value of $34!/10^7$ modulo 10 directly.

$a = 2$ **with reasoning [2]**

Although the modulo 8 approach is more slick, a candidate who successfully applied the modulo 10 method – or any other valid method of calculating the value of a – would receive full marks for this part of the question. *If a solution is correct, and all reasoning is explained, then the solution will receive full marks, regardless of how elegant/inelegant or short/long it is.*

In order to calculate the values of c and d, it is necessary to consider various factors of 34!. In particular, from the fact that 34! is divisible by 9 and 11 it follows that $c+d-3$ is divisible by 9 and that $c-d-8$ is divisible by 11. From these two equations, it is easy to deduce that $c = 0, d = 3$.

$c = 0, d = 3$ **with reasoning [6]**

In order to prove that $c + d - 3$ is divisible by 9, one uses the fact that an integer is divisible by 9 only if the sum of its decimal digits is divisible by 9. If a candidate added up the long string of digits in the question incorrectly, she might erroneously deduce that $c+d-5$, say, was divisible by 9. However, if she went on to complete the solution otherwise correctly, no deduction would be made.

A single arithmetic slip [no deduction]

Trivial arithmetic errors are not heavily penalised at the BMO. Candidates should not, however, regard this as a licence to be careless in their calculations!

Two or more arithmetic slips [−1]

If a candidate calculated b, say, wrongly, but then went on to work out c and d by a method which failed to work only because of her erroneous value for b, she would receive full marks for her calculation of c and d.

2. **The triangle ABC, where $AB < AC$, has circumcircle S. The perpendicular from A to BC meets S again at P. The point X lies on the line segment AC, and BX meets S again at Q. Show that $BX = CX$ if and only if PQ is a diameter of S.**

   When the examiners set a markscheme for the BMO paper, they try to ensure that candidates who manage to solve questions completely are rewarded for doing so. As the rubric for the paper says: *one complete solution will gain far more credit than several unfinished attempts.* A question such as this one is often given a $0^+/10^-$ markscheme. This is really two separate markschemes: one scheme for scripts in which the candidate has essentially solved the question fully (possibly with some small omissions or errors), and one scheme for scripts in which the candidate has not really cracked the problem. The examiners first decide which group a candidate's work on a given question belongs to, and then mark that work according to the appropriate markscheme.

   - 10^- **markscheme**

     A complete solution [10]

     A single mathematical error [−2]

     A candidate might be tempted to start a solution by noting that $\angle CAP = \angle CBP$, these being angles in the same segment. However, if B is obtuse, A and B lie on opposite sides of the line CP, and the candidate's claim is wrong. If her proof is otherwise correct, and can be easily repaired to cover the case in which B is obtuse, an error of this sort would be penalised 2 marks.

     One direction of 'if and only if' implication omitted [−3]

Candidates in the BMO should be very careful in their use of logical implications. In this problem, two things are to be proved: that if $BX = CX$ then PQ is a diameter of S; and that if PQ is a diameter of S then $BX = CX$. A candidate who proved only one of these would receive 7 marks.

- 0^+ **markscheme**

  Nontrivial progress towards a proof of one part of the problem [up to 2]

  The examiners would decide in detail in advance which observations were worthy of one or both of these marks. They might, for example, decide: that

  $$BX = CX \Leftrightarrow \angle XBC = \angle XCB$$

  is worth no marks; that

  $$BX = CX \Leftrightarrow \angle XBC = \angle XQA$$

  is worth 1 mark; and that

  $$BX = CX \Leftrightarrow BC \| AQ$$

  is worth 2 marks.

BMO markschemes are often very complicated, in order to take account of many different possible solutions to a problem. To summarise the above, however, the following points always hold:

- A candidate will receive full marks for a question only if all her reasoning is carefully explained.
- Elegance and brevity of solutions are admirable, but do not receive any extra credit in marking.
- Trivial arithmetic errors will not be heavily penalised.
- Progress towards a solution will always be rewarded; but in order to achieve high marks, candidates should aim to produce a small number of complete solutions, rather than a larger number of partial attempts.