

Further International Selection Test
May 17, 1972
Time 3 Hours
Only your best 3 answers will score.

- 1 Show how to assign to the vertices of a regular polygon with 2^n vertices, numbers such that
 - (a) only the digits 1 and 2 are used
 - (b) each number has n digits
 - (c) each vertex has a different number, and
 - (d) neighbouring vertices have numbers differing in one and only one digit place.

- 2 a, b, c, d are positive numbers and
$$S = \frac{a+c}{a+b} + \frac{b+d}{b+c} + \frac{c+a}{c+d} + \frac{d+b}{d+a} .$$
Prove that S is not less than 4, and obtain necessary conditions for $S=4$.

3. There are n persons present at a meeting. Every two persons are either friends of each other or strangers to each other. No two friends have a friend in common. Every two strangers have two and only two friends in common.
Prove that each person has the same number of friends at the meeting. If this number is 5, find n .

4. When $k = 1$ find all points P in space such that
$$a \cdot PA^2 + b \cdot PB^2 + c \cdot PC^2 = kabc ,$$
where a, b, c are the lengths of the sides BC, CA, AB of triangle ABC , and prove your result.
What is the effect of altering k ?