a, b, c, d are different positive real numbers. Prove that if at least one of the numbers c and d lies between the numbers a and b, or at least one of the numbers a and b lies between the numbers c and d, then

\[ \sqrt{(a + b)(c + d)} \geq \sqrt{ab} + \sqrt{cd}. \]

Otherwise show that the four numbers can be chosen so that (*) is false.

2. Two equilateral triangles have a common vertex C. Going round each triangle in the anti-clockwise direction the vertices are lettered C, A, B and C, A', B'. O is the centre of triangle CAB and neither A' nor B' coincides with O. M is the midpoint of A'B and N is the midpoint of AB'. Prove that triangles OB'M and OA'N are similar.

3. The sequence of positive integers \( a_n \) is defined by

\[ a_0 = 1979, \quad a_{n+1} = \left[ \sqrt{a_0 + a_1 + \ldots + a_n} \right] \quad (n \geq 0), \]

where \([x]\) denotes the greatest integer not greater than \( x \). (For example \([3]\) = 3 and \([5]\) = 5.) Determine \( a_{1979} \).

4. Let \( b(k) \) denote the number of 1's in the binary expansion of the non-negative integer \( k \). For example \( b(13) = 3 \) since 13 is 1101 in binary notation. Prove that for all positive integers \( n \),

\[ \sum_{k=0}^{2^{n-1}} (-1)^{b(k)} k^n = (-1)^n 2^{\frac{1}{2} n(n-1)} (n!) . \]