1. Prove that if the polynomial
   \[ p(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-1}x + a_n, \]
   whose coefficients are integers, takes the value 1990 for four distinct integer values of \( x \), then it cannot take the value 1997 for any integer value of \( x \).

2. The integer part \([x]\) of a number \( x \) is the greatest integer which is not greater than \( x \). The fractional part \( f(x) \) is defined by \( f(x) = x - [x] \).
   Find a positive number \( x \) such that
   \[ f(x) + f\left(\frac{1}{x}\right) = 1. \]
   Are there any rational solutions?

3. State the cosine rule for a triangle.
   Prove that for arbitrary positive real numbers \( a, b, c \),
   \[ \sqrt{a^2 + b^2 - ab} + \sqrt{b^2 + c^2 - bc} \geq \sqrt{a^2 + c^2 + ac}. \]

4. Let \( d \) denote the length of the smallest diagonal of all rectangles inscribed in a triangle \( T \). (By inscribed we mean that all vertices of the rectangle lie on the boundary of \( T \)).
   Determine the maximum value of \( d^2/\text{area}(T) \) taken over all triangles.

5. \( I \) is the centre of the circle inscribed to triangle \( ABC \); \( J \) is the centre of the escribed circle which touches \( AB \) and \( AC \) produced beyond \( B \) and \( C \) respectively.
   Prove that
   \[ AI.AJ = AB.AC \]
   and that
   \[ AI.BJ.CJ = AJ.BI.CI. \]