Q1 Let \( x_1, x_2, \ldots, x_n \) be real numbers such that \( 0 \leq x_i \leq 2 \) for each \( i \). Prove that
\[
\sum_{i=1}^{n} \sum_{j=1}^{n} |x_i - x_j| \leq n^2.
\]
When does equality hold?

Q2 The in-circle of \( \triangle ABC \), where \( AB \parallel AC \), touches \( BC \) at \( L \), and \( LM \) is a diameter of the in-circle. \( AM \) produced cuts \( BC \) at \( N \).
(i) Prove \( NL = AB - AC \).
(ii) A circle \( S \) of variable radius touches \( BC \) at \( M \). The tangents (other than \( BC \)) from \( B \) and \( C \) to \( S \) intersect at \( P \). \( P \) moves as the radius of \( S \) varies. Find the locus of \( P \).

Q3 The sequence \( u_n \) is defined for positive integers by
\[
u_1 = 1, \quad u_{n+1} = u_n + \lfloor \sqrt{u_n} \rfloor, \quad (n \geq 1).
\]
Here \( \lfloor x \rfloor \) denotes the nearest integer to \( x \), i.e. the integer \( M \) such that \( x - \frac{1}{2} \leq M < x + \frac{1}{2} \).
Determine, with proof, the final (i.e. rightmost) digit of the integer \( u_{1985} \).

Q4 Let \( A, B, C, D \) be points on a sphere of radius 1. Given that
\[AB \cdot BC \cdot CA \cdot DA \cdot DB \cdot DC = \frac{512}{27},\]
prove ABCD a regular tetrahedron.

Q5 Let \( B_n \) be the number of ways of partitioning a set with \( n \) elements, i.e. expressing it as the union of one or more non-empty subsets, no two of which have a common element. Eg. \( B_3 = 5 \), the partitionings of \( \{abc \} \) being
\[a,b,c \quad a,bc \quad b,ac \quad c,ab \quad abc\]
Let \( C_n \) be the number of partitionings in which each subset has more than one element, e.g. \( C_3 = 1 \). Prove that for \( n \geq 1 \)
\[
C_n = B_{n-1} - B_{n-2} + B_{n-3} - \ldots + (-1)^n B_1.
\]

Q6 Solve in non-negative integers \( x, y, z \) the equation
\[5^x \cdot 7^y + 4 = 3^z.\]