British Mathematical Olympiad Committee

FINAL SELECTION TEST

Sunday 5th April 1992

Time allowed : $4\frac{1}{2}$ hours

1. Circles C_1, C_2, with centres O_1, O_2 and radii r_1, r_2 respectively, are drawn so that the distance between their centres is given by $O_1O_2 = \sqrt{r_1^2 + r_2^2}$. The circles C_1 and C_2 intersect at A and B. From the point P on C_1 furthest from O_2 lines PA, PB are drawn to intersect C_2 at D, E respectively. Prove that DE is the diameter of C_2 perpendicular to O_1O_2.
   A point Q (distinct from P) is now chosen on the major arc AB of C_1. Lines QA, QB are drawn to intersect C_2 at F, G respectively. Prove that FG is also a diameter of C_2. Locate, with proof, the point where FB and GA meet.

2. Let a_n be the last non-zero digit in the decimal representation of $n!$. Does the sequence $a_1, a_2, a_3, \ldots$ become periodic after a finite number of terms?

3. An ancient dwarfish ritual requires ten dwarves to stand on level ground as the sun rises on Midsummers's Day, so that

   ($\star$) *among every subset of five dwarves some four lie on a circle.*

   Among all such arrangements, find the minimum value of the maximum number of dwarves who lie on a single circle.
   What is the corresponding result for the arrangements of N dwarves satisfying ($\star$)?