

FINAL SELECTION TEST

SUNDAY 31 MARCH 1996

Time Allowed: $4\frac{1}{2}$ hours

1. Two circles Γ_1 and Γ_2 intersect at D and E .

The point B on Γ_1 and the point C on Γ_2 are such that D is the mid-point of BC .

The tangent to Γ_1 at B and the tangent to Γ_2 at C meet at A .

Prove that $DA \cdot DE = DC^2$.

2. Let \mathbf{N} denote the set of all positive integers.

Show that there is a unique function $f : \mathbf{N} \rightarrow \mathbf{N}$ satisfying

$$f(m + f(n)) = n + f(m + 95)$$

for all m, n in \mathbf{N} .

What is the value of $f(1) + f(2) + \dots + f(19)$?

3. One of the most important festivals in the Dwarfish calendar is the ancient feast of "Dozens" at which the $12k$ members of the Dwarfish Council of Elders meet and exchange presents (and maybe share the odd glass of mead).

This ritual is governed by two rules.

Firstly, each of the $12k$ Elders must exchange presents with exactly $3k + 6$ other Elders. Secondly, given any pair of Elders, the number of Elders who exchange presents with both must be constant (although the value of this magic constant is not revealed).

How big must the Dwarfish Council of Elders be?