**FINAL SELECTION TEST**

**SUNDAY 13 APRIL 1997**

**Time allowed: $4\frac{1}{2}$ hours**

1. A finite sequence of integers $a_0, a_1, ..., a_n$ is called *quadratic* if

$$|a_i - a_{i-1}| = i^2 \quad \text{for} \quad i = 1, 2, ..., n.$$

(i) Prove that, for any two integers $b$ and $c$, there exist a positive integer $n$ and a quadratic sequence $a_0, a_1, ..., a_n$ with

$$a_0 = b \quad \text{and} \quad a_n = c.$$

(ii) Find the smallest positive integer $n$ for which there exists a quadratic sequence $a_0, a_1, ..., a_n$ with

$$a_0 = 0 \quad \text{and} \quad a_n = 1997.$$

2. Determine whether or not there exist two disjoint infinite sets $\mathcal{A}$ and $\mathcal{B}$ of points in the plane satisfying the following two conditions:

(a) no three points in $\mathcal{A} \cup \mathcal{B}$ are collinear, and the distance between any two points in $\mathcal{A} \cup \mathcal{B}$ is at least 1;

(b) there is a point of $\mathcal{A}$ in any triangle whose vertices are in $\mathcal{B}$, and there is a point of $\mathcal{B}$ in any triangle whose vertices are in $\mathcal{A}$.

3. Let $ABC$ be an acute-angled triangle with $BC > CA$.

Let $O$ be the circumcentre, $H$ the orthocentre and $F$ the foot of the altitude $CH$ of $\Delta ABC$.

Let the perpendicular to $OF$ at $F$ meet the side $CA$ at $P$.

Prove that $\angle FHP = \angle BAC$.

What happens when $BC \leq CA$, but the triangle is still acute-angled?