FINAL SELECTION TEST

SUNDAY 5 APRIL 1998

Time allowed: $4\frac{1}{2}$ hours

1. Let $P(x)$ be a polynomial with real coefficients such that

$$P(x) > 0 \qquad \text{for all} \quad x \geq 0.$$

Prove that there exists a positive integer n such that $(1+x)^n P(x)$ is a polynomial all of whose coefficients are non-negative.

2. The altitudes through the vertices A, B and C of an acute-angled triangle ABC meet the opposite sides at D, E and F respectively.

The line through D parallel to EF meets the lines AC and AB (extended if necessary) at Q and R, respectively.

The line through E and F meets the line through B and C at P.

Prove that the circumcircle of ΔPQR passes through the mid-point of BC.

3. An infinite arithmetic progression, whose terms are all positive integers, contains

(i) a perfect square which is not a perfect cube, and

(ii) a perfect cube which is not a perfect square.

Prove that the arithmetic progression contains a perfect sixth power.