1. A sequence of positive integers $a_1, a_2, \ldots$ is defined as follows: $a_1 = 1$ and, for $n \geq 1$, $a_{n+1}$ is the smallest integer greater than $a_n$ such that for any $i, j, k$ (not necessarily distinct) in $\{1, \ldots, n + 1\}$ we have $a_i + a_j \neq 3a_k$. Determine $a_{1999}$.

2. Let $ABCD$ be a cyclic quadrilateral. Let $E$ and $F$ be points on the sides $AB$ and $CD$ respectively such that $AE : EB = CF : FD$. Let $P$ be a point on the segment $EF$ such that $PE : PF = AB : CD$. Prove that $P$ is equidistant from the lines $AD$ and $BC$.

3. Ten points are marked in the plane, no three of which are collinear. Each pair of points is connected by a segment. Each segment is given one of $k$ colours, in such a way that, for any $k$ of the points, there exist $k$ segments, each joining two of those points and no two of the same colour. Determine the smallest positive integer $k$ for which this is possible.