FINAL SELECTION TEST

SUNDAY 11 APRIL 1999

08.15-12.45

1. A sequence of positive integers $a_1, a_2, \ldots$ is defined as follows: $a_1 = 1$ and, for $n \geq 1$, a_{n+1} is the smallest integer greater than a_n such that for any i, j, k (not necessarily distinct) in $\{1, \ldots, n+1\}$ we have $a_i + a_j \neq 3a_k$. Determine a_{1999}.

2. Let $ABCD$ be a cyclic quadrilateral. Let E and F be points on the sides AB and CD respectively such that $AE : EB = CF : FD$. Let P be a point on the segment EF such that $PE : PF = AB : CD$. Prove that P is equidistant from the lines AD and BC.

3. Ten points are marked in the plane, no three of which are collinear. Each pair of points is connected by a segment. Each segment is given one of k colours, in such a way that, for any k of the points, there exist k segments, each joining two of those points and no two of the same colour. Determine the smallest positive integer k for which this is possible.