

FINAL SELECTION TEST

SUNDAY 9 APRIL 2000

08.15-12.45

1. Let n be a positive integer that is not a multiple of 3. Show that, for any integer $k \geq n$, there is a multiple of n having digit sum exactly k .

2. Let ABC be a triangle, and let M be a point inside the triangle. Prove that

$$MA + MB + MC + \min(MA, MB, MC) < AB + BC + CA.$$

3. Let $n \geq 2$ be a fixed positive integer. For any $n \times n$ matrix with distinct positive entries, we define its *characteristic* as follows. For any two entries in the same row or column, we calculate the ratio of the larger to the smaller. The characteristic of the matrix is then the smallest of these $n^2(n-1)$ fractions. What is the greatest possible characteristic of an $n \times n$ matrix whose entries are (in some order) $1, 2, 3, \dots, n^2$?