FINAL SELECTION TEST

WEDNESDAY 11 APRIL 2001

08.15-12.45

1. What is the smallest number of squares it is possible to mark on a $5n \times 5n$ chessboard in such a way that no row or column contains a block of $3n$ consecutive unmarked squares?

2. Prove that there exists a polynomial $P(x)$ with integer coefficients such that the numbers $P(1), P(2), P(3), \ldots, P(2001)$ are distinct powers of 2.

3. The tangents at B and A to the circumcircle of the acute-angled triangle ABC meet the tangent at C at T and U respectively. Lines AT and BC meet at P, and Q is the midpoint of AP; lines BU and AC meet at R, and S is the midpoint of BR. Prove that the angles ABQ and BAS are equal, and determine (in terms of ratios of side-lengths) the triangles for which this angle is maximised.