First Selection Test

April 2003

1. Consider triangle ABC. Let U, V, W be points such that U is on the line through B and C, V is on the line through C and A, W is on the line through A and B. It is given that AU, BV and CW are concurrent at a point P. Also AU is a median of the triangle, BV is an altitude and CW is the internal angle bisector of $\angle BCA$. Suppose that P lies on the perpendicular bisector of at least one of the sides of triangle ABC. Prove that triangle ABC is equilateral.

2. Find all positive integers n such that the equation

$$x + y + u + v = n\sqrt{xyuv}$$

   has a positive integer solution x, y, u, v.

3. Suppose that m, n are positive integers with $m < 2002$ and $n < 2003$. We are given 2002×2003 distinct real numbers. These real numbers are entered into the 1×1 cells of a 2002×2003 rectangular "chessboard" which has 2002 rows and 2003 columns with exactly one number in each cell. A little square is called "feeble" if the number it contains is simultaneously less than at least m numbers written in cells in the same column, and less than at least n numbers written in cells in the same row. Let there be s feeble squares for a given way of entering the numbers. Minimize s (as a function of m and n) over all possible ways of entering the numbers.