UK IMO FST1

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1. An infinite sequence $a_0, a_1, a_2, \ldots$ of real numbers satisfies the condition $a_n = |a_{n+1} - a_{n+2}|$ for every $n \geq 0$ with a_0, a_1 positive and distinct. Can this sequence be bounded?

2. Let $\tau(n)$ denote the number of positive divisors of the positive integer n. Prove that there are infinitely many positive integers a such that $\tau(an) = n$ has no positive integer solution n.

3. Let P be a convex polygon. Prove that there is a convex hexagon which is contained in P and which occupies at least 75% of the area of P.