

# FST1 2006

Trinity College Cambridge

1. Let  $E$  be the intersection of the diagonals of the cyclic quadrilateral  $ABCD$ . Let  $F$  and  $G$  denote the respective midpoints of the sides  $AB$  and  $CD$ . Prove that the line through  $G$  which is perpendicular to  $AC$ , the line through  $F$  which is perpendicular to  $BD$  and the line through  $E$  which is perpendicular to  $AD$  are concurrent.
2. Let  $S = \{1, 2, 3, \dots, 2006\}$ . Prove that there are 14 subsets of  $S$  which have the following property: if  $s \in S$ , then there are 7 of these 14 sets which have  $s$  as their only common element.
3. Does there exist an infinite subset  $T$  of the positive integers such that whenever  $a, b$  are in  $T$ , then  $a^2 - ab + b^2$  divides  $a^2b^2$ ?