First Selection Test: Exam 2

IMO camp, Trinity College Cambridge

2-iv-2007

Problem 1 Let a, b be positive integers such that for every positive integer n we have $a^n + n \mid b^n + n$. Prove that $a = b$.

Problem 2 Let $\mathbb{R}^+$ denote the set of positive real numbers. Find all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that

$$x^2 \left(f(x) + f(y)\right) = (x + y) f(f(x)y)$$

for all $x, y \in \mathbb{R}^+$.

Problem 3 Let A be a point exterior to a circle Γ. Two lines through A meet Γ at B, C and D, E respectively, with D between A and E. Draw the line through D which is parallel to AC, and let it meet Γ again at F. Suppose that AF meets Γ again at G, and that EG meets AC at M. Prove that

$$\frac{1}{AM} = \frac{1}{AB} + \frac{1}{AC}.$$

Time allowed: 4 hours 30 minutes