4. Find all solutions to
\[ p(p + 1) + q(q + 1) = n(n + 1) \]
where \( p \) and \( q \) are prime numbers and \( n \) is a positive integer.

5. Let \( S \) be a set of 1953 points in the plane. Every two points of \( S \) are at least distance 1 apart. Prove that \( S \) contains a subset \( T \) of 217 points, every two at least distance \( \sqrt{3} \) apart.

6. The monic polynomial
\[ P(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 \]
of degree \( n > 1 \) has \( n \) distinct negative real roots. Prove that
\[ a_1P(1) > 2n^2a_0. \]

Each question is worth seven marks.

Time: 4 hours, 30 minutes.