

First Selection Test: Paper 2

Trinity College, Cambridge

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1. Let $ABCD$ be a cyclic quadrilateral whose diagonals AC and BD meet at E . The extensions of the sides AD and BC beyond A and B meet at F . Let G be the point such that $ECGD$ is a parallelogram, and let H be the image of E under reflection in AD . Prove that D, H, F, G are concyclic.
2. For sets of numbers X and Y , the set $X + Y$ is the set of sums $x + y$, where $x \in X$ and $y \in Y$.
 - (a) Does there exist a partition of the set \mathbb{Z} into three non-empty sets A, B , and C , such that the sets $A + B, B + C, C + A$ are pairwise disjoint?
 - (b) Does there exist a partition of the set \mathbb{Q} into three non-empty sets A, B , and C , such that the sets $A + B, B + C, C + A$ are pairwise disjoint?
3. Let x and y be positive integers. If $x^{2^n} - 1$ is divisible by $2^n y + 1$ for every positive integer n , prove that $x = 1$.