FST2

30.3.15

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4. An acute-angled triangle $\triangle ABC$ is given, and A_1, B_1, C_1 are the midpoints of sides BC, CA, AB respectively. The internal angle bisector of $\angle AC_1C$ meets AC at L, and the internal angle bisector of $\angle CC_1B$ meets BC at K. The line LK intersects B_1C_1 at A_2, and A_1C_1 at B_2. Prove that the lines AA_2, BB_2, CC_1 are concurrent.

5. Let $n > 1$ be a given integer. Define

$$a_k := \left\lfloor \frac{n^k}{k} \right\rfloor, \quad \text{for each } k \geq 1.$$

Prove that infinitely many terms of the sequence (a_k) are odd.

[For a real number x, $\lfloor x \rfloor$ is the largest integer not greater than x.]

6. Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ such that

$$f\left(2m + f(m) + f(m)f(n)\right) = nf(m) + m$$

for all $m, n \in \mathbb{Z}$.