

Wednesday, July 9, 2014

Problem 4. Points P and Q lie on side BC of acute-angled triangle ABC so that $\angle PAB = \angle BCA$ and $\angle CAQ = \angle ABC$. Points M and N lie on lines AP and AQ , respectively, such that P is the midpoint of AM , and Q is the midpoint of AN . Prove that lines BM and CN intersect on the circumcircle of triangle ABC .

Problem 5. For each positive integer n , the Bank of Cape Town issues coins of denomination $\frac{1}{n}$. Given a finite collection of such coins (of not necessarily different denominations) with total value at most $99 + \frac{1}{2}$, prove that it is possible to split this collection into 100 or fewer groups, such that each group has total value at most 1.

Problem 6. A set of lines in the plane is in *general position* if no two are parallel and no three pass through the same point. A set of lines in general position cuts the plane into regions, some of which have finite area; we call these its *finite regions*. Prove that for all sufficiently large n , in any set of n lines in general position it is possible to colour at least \sqrt{n} of the lines blue in such a way that none of its finite regions has a completely blue boundary.

Note: Results with \sqrt{n} replaced by $c\sqrt{n}$ will be awarded points depending on the value of the constant c .