UK Maths Trust

Mathematical Competition for Girls

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Solutions

Although in this paper you were only required to provide numerical answers, we present full solutions, including all the reasoning leading to those answers. These will be helpful if you ever enter an olympiad which requires full solutions.

The answers that needed to be entered on the answer sheets are given at the start of each solution.

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions. They are not intended to be the 'best' possible solutions; in some cases we have suggested alternatives, but readers may come up with other equally good ideas.

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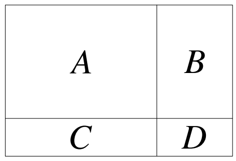
1. A rectangular apartment is divided into four rectangular rooms, by two straight interior walls which are parallel to the exterior walls. The rooms have areas A, B, C and D metres squared.



(a) Find the value of D when $A = 8$, $B = 4$ and $C = 12$. [2 marks]

(b) Find the value of C when $A = 7.5$, $B = 6.5$ and $D = 26$. [2 marks]

(c) In this part of the question, A, B, C and D have the values found in part (a). Additionally, the length and the width of each room are whole numbers.

Let P be the perimeter of the apartment. Find all possible values of P.

Enter the following on the answer sheet:

(i) The smallest possible value of P. [3 marks]

(ii) The largest possible value of P. [3 marks]

COMMENTARY

You could write some equations to find the side lengths of each room. However, for the first three parts, it is simpler to consider ratios of areas.

Rooms A and B have the same width, so their areas are in the same ratio as their length. The same is true for rooms C and D. It follows that $A : B = C : D$.

(a) $8 : 4 = 12 : D$, so $D = 6$.

(b) $7.5 : 6.5 = C : 26$ so $C = 30$.

(c) Denote the lengths of A and B by x and y, and the widths of A and C by z and t, respectively. Then the area of B gives $yz = 4$, so there are three options: $(y, z) = (4, 1), (2, 2)$ or $(1, 4)$.

The first case gives $t = 1.5$ which is impossible.

In the second case, $x = 4, t = 3$ so $P = 2(6 + 5) = 22$.

In the third case, $x = 2, t = 6$ so $P = 2(3 + 10) = 26$.

Answer: (a) 006 (b) 030 (c) (i) 022 (ii) 026

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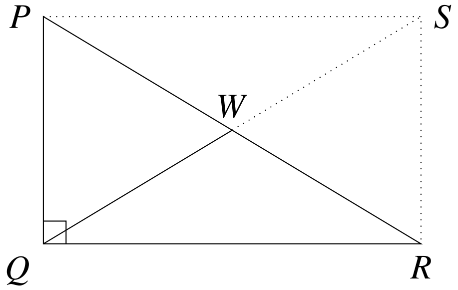
2. (a) Triangle PQR has a right angle at Q and W is the midpoint of the side PR. Given that $PQ = 6$ and $QR = 8$, use the diagram below to find the length of QW.

[2 marks]



(b) Triangle ABC has a right angle at B and $AB < BC$. Let M be the midpoint of AC. Let D be the reflection of C in the line BM (extended beyond M).

Given that angle DAC is $56°$ find the size of the angle

(i) ACD, [4 marks]

(ii) BAC. [4 marks]

Commentary

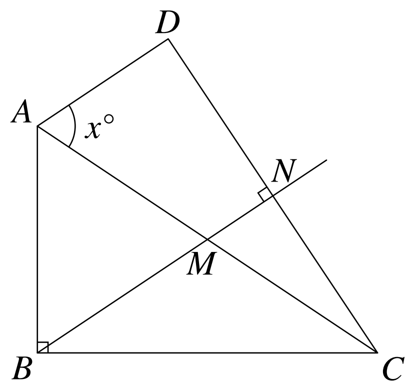
Start by drawing a clear diagram and identify the angle you know and the angle you want to find.

Angle BAC can be found from triangle ABM, and part (a) tells us something important about it. You are given angle DAC; what can you say about the line AD? Think about how you can sue properties of reflection.

(a) The diagonals of a rectangle are equal and bisect each other. Hence $QW = \frac{1}{2}QS = \frac{1}{2}PR$.

But, by Pythagoras, $PR^2 = PQ^2 + QR^2 = 100$ so $PR = 10$ and $QW = 5$.

(b) We are going to answer a more general question, finding the size of the angles ACD and BAC in terms of angle DAC. So let angle $DAC = x°$.



Let N be the intersection of CD and BM. By the properties of reflection, N is the midpoint of BD and BNC is a right angle.

In triangle ACD, M and N are midpoints of AC and CD, so MN is parallel to AD. It follows that ADC is a right angle and angle $NMC = x°$.

Angles in the right angled triangle MCN add up to $180°$, so the size of the angle ACD is $90 - x = 34°$.

Using the result of part (a), $BM = MC = MA$ so triangle AMB is isosceles with angle

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angle x° at M. Since angles in a triangle add up to 180°, we find that angle MAB equals $\left(\dfrac{180 - x}{2}\right)^\circ = 62^\circ$.

Answer: (a) 005 (b) (i) 034 (ii) 062

3. For any real number x, the *integer part* of x, $\lfloor x \rfloor$, is the largest whole number which is smaller than or equal to x. For example, $\lfloor 2.3 \rfloor = 2$, $\lfloor 2.8 \rfloor = 2$ and $\lfloor 2 \rfloor = 2$.

(a) Find all positive integers a such that $\left\lfloor \frac{a}{5} \right\rfloor = 7$.

Enter the following on the answer sheet:

(i) The number of possible values of a. [1 mark]

(ii) The smallest possible value of a. [1 mark]

(iii) The largest possible value of a. [2 marks]

(b) Find all positive integers b such that $\left\lfloor \frac{b}{3} \right\rfloor - \left\lfloor \frac{b}{5} \right\rfloor = 3$.

Enter the following on the answer sheet:

(i) The number of possible values of b. [2 marks]

(ii) The smallest possible value of b. [2 marks]

(iii) The largest possible value of b. [2 marks]

Commentary

This question may look intimidating because of the unfamiliar notation. Start by trying some numbers to understand what the integer part function does.

To answer part (b), it is possible to roughly estimate b and then test some numbers around that value. However, it is better to develop and algebraic method which will ensure that you do not miss any of the answers. This method relies on observing that, for any real number x, if $\lfloor x \rfloor = n$ then $n \le x < n + 1$. We will use this method in both parts of the question.

(a) For any real number x, if $\lfloor x \rfloor = n$ then $n \le x < n + 1$. Hence we have

$$7 \le \frac{a}{5} < 8,$$

$$35 \le a < 40.$$

Since a is an integer, the possible values of a are $35, 36, 37, 38$ and 39.

(b) Suppose $\left\lfloor \frac{b}{3} \right\rfloor = m$ and $\left\lfloor \frac{b}{5} \right\rfloor = n$. Then $m \le \frac{b}{3} < m + 1$ so $3m \le b < 3m + 3$ and, similarly, $5n \le b < 5n + 5$.

On the other hand, we know that $m - n = 3$ so we can replace m by $n + 3$ in the first inequality to get $3n + 9 \le b < 3n + 12$. Thus b has to satisfy both this inequality and $5n \le b < 5n + 5$.

For this to be possible, the two intervals have to overlap. It is probably easier to find when they *don't* overlap, which is when either $3n + 12 \le 5n$ or $5n + 5 \le 3n + 9$. (You may want to draw a number line to see why this is the case.)

Solving the two inequalities gives that the values that n *cannot* be are $n \leq 2$ and $n \geq 6$, so the only possible values of n are $3, 4$ and 5, leading to $18 \leq b \leq 26$.

Unfortunately, not all the integer values of b in this interval work, so we need to check all of them. We find that the possible values of b are $18, 19, 21, 22, 23, 25$ and 26.

Answer: (a) (i) 005 (ii) 035 (iii) 039; (b) (i) 007 (ii) 018 (iii) 026

4. (a) You have three red and four blue cubes. The cubes of the same colour are identical.

This part of the question is about the number of ways to distribute all seven cubes among some boxes, following these rules:

- A box may be empty (so it is possible to put all seven cubes in one box).
- The boxes are considered different (so putting all seven cubes in the first box counts as a different solution from putting all seven cubes in the second box).

(i) How many ways are there to distribute all seven cubes among two boxes? [2 marks]

(ii) How many ways are there to distribute all seven cubes among three boxes, so that no colour appears in all three boxes? [3 marks]

(b) The *factorial* of a positive integer n is defined as $n! = 1 \times 2 \times 3 \times \cdots \times n$. For example, $3! = 1 \times 2 \times 3 = 6$.

In how many ways can we write $10! = abc$, where a, b, c are positive integers, such that there is no factor greater than one which is common to all three of them?

Reorderings of a, b, c count as different solutions. For example, $120 \times 54 \times 560$ is not allowed because all three numbers share a common factor of 2, while $640 \times 162 \times 35$ is allowed and counts as a different solution from $162 \times 640 \times 35$.

Note: This question part may have a four digit answer. [5 marks]

Commentary

In part (a), the most important thing to realise is that the two colours can be distributed independently. When you have three boxes you need to think carefully how you are going to ensure that only one or two boxes contain each colour.

Before starting part (b), think about how part (a) can help. You are trying to write a number as a product, so you are trying to distribute all the factors you have among the "boxes" a, b and c. You should find that you can count in exactly the same way as in part (a).

(a) We count the number of ways to distribute the cubes of each colour separately. Those numbers then need to be multiplied because for every distribution of red cubes we can have every possible distribution of blue cubes.

(i) The three red cubes can be split as $0 + 3, 1 + 2, 2 + 1$ or $3 + 0$, which is 4 ways. Similarly, the blue cubes can be split in five ways. Hence the total number of ways to distribute the cubes is $4 \times 5 = 20$.

(ii) There are 3 ways for all the red cubes to be in the same box. Otherwise one box contains no red cubes, and there are 2 ways to distribute them between the other two boxes ($1 + 2$ or $2 + 1$). There are three ways to select the box with no red cubes. Hence the number of ways to distribute the red cubes is $3 + (3 \times 2) = 9$.

Similarly, the number of ways to distribute the blue cubes is $3 + (3 \times 3) = 12$.

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Hence the total number of ways to distribute the cubes is $9 \times 12 = 108$.

(b) We can factorise $10! = 2^8 \times 3^4 \times 5^2 \times 7$. Each of the numbers a, b and c is a product of some powers of $2, 3, 5$ and 7, where some of the powers must be zero to avoid all three numbers having a common factor greater than one.

The factor of 7 can be in exactly one of the three numbers, giving three options.

Either one of the three numbers is a multiple of 25, or two of them are multiples of 5, giving $3 + 3 = 6$ options.

Distributing the factors of 3 among the three numbers is the same as distributing the four blue cubes in part (a)(ii), which is 12 options.

Finally, we can find the way to distribute the factors of 2 in the same way (imagine there are eight orange cubes), giving $3 + (3 \times 7) = 24$ options.

Therefore the total number of choices for the values of a, b and c is $3 \times 6 \times 12 \times 24 = 5184$.

$\boxed{\text{Answer: (a) (i) 020 (ii) 108; (b) 5184}}$

5. This question is about finding winning strategies in two-player games. We say that a player has a *winning strategy* if they can win the game no matter what moves the other player makes.

(a) In the first game, Players A and B start with a pile of 100 counters and take turns to remove either 2 or 3 counters from the pile. Player A goes first. The winner is the player who removes the final counter. If only one counter remains, the result is a draw.

Work out the winning strategy for Player B and then answer this question:

At three different points in the game, there are 98, 73 and 47 counters at the start of Player B's turn. On the answer sheet, enter the number of counters player B should remove in each case.

(For example, if you think that they should remove 2 counters in each case, enter 222 as your answer.) [3 marks]

(b) In the second game, Players A and B start with a pile of N counters and take turns to remove 1, 3 or 4 counters from the pile. Player A goes first. The winner is the player who removes the final counter.

(Note: If there are only two counters left, the current player must remove one counter.)

Parts (i) and (ii) each give a value of N for which Player B has a winning strategy. In each part, the three-digit answer is the number of counters Player B should remove on their first turn in response to Player A removing 1, 3 and 4 counters, respectively.

(For example, if you think that Player B should remove 1 counter when Player A removes 1, 1 counter when Player A removes 3, and 3 counters when Player A removes 4, your answer should be 113.)

(i) $N = 7$ [1 mark]

(ii) $N = 9$ [1 mark]

Note: There may be more than one correct answer. You only need to choose one of them.

(iii) For how many values of N between 100 and 200, inclusive, does Player B have a winning strategy? [5 marks]

COMMENTARY

The best way to tackle problems like this is to try playing the game. It is useful to think "from the end": which numbers of counters would the winning player reach on their penultimate move?

These "winning positions" should not be too difficult to find for the first game. In the second game, parts (i) and (ii) suggest that there are two types of winning positions. Think about the winning positions you discovered in parts (i) and (ii): they are 0, 2, 7 and 9. Can you guess what the other winning positions are? You can continue going "backwards" to find a few more if you are not sure.

(a) Notice that the number of counters at the start is a multiple of 5, as is the final number (0). So Player B can win by ensuring that, at the end of their turn, the number of counters is always a multiple of 5.

They can achieve this by always making the "opposite" move from Player A: whenever Player A removes two counters, Player B should remove three, and vice versa. This also ensures that Player A can never end on a multiple of 5.

Since the number of counters decreases every turn, Player B will eventually reach 0.

(b) (i) Note that if Player B ends their turn with 2 counters remaining, they are guaranteed to win (because Player A must remove 1). Player B also obviously wins if they reach 0 counters.

When Player A starts with 7 counters, Player B should aim to end their first turn on 0 or 2. If Player A's first move is to remove 1 counter, Player B cannot reach 0 in one turn, so has to end on 2. Otherwise they can end on 0 straight away. Therefore, in response to Player A removing 1, 3 and 4 counters, Player B should respond by removing 4, 4 and 3, respectively.

(ii) When Player A starts with with 9 counters, Player B cannot reach 0 in one turn, but they can reach 2 or 7. From the previous part, we know that these are both winning positions, so Player B should remove 1, 4 and 3 counters, respectively.

(iii) Player B's winning strategy is to ensure that, at the *end* of each of their turns, the number of counters is either a multiple of 7 or two more than a multiple of 7.

They can carry out this strategy as follows. If Player A starts on a multiple of 7, Player B can ensure that the number of counters decreases by 5 or 7 (by taking 4, 4 and 3 counters after Player A takes 1, 3 and 4, respectively). If Player A starts on a number which is 2 more than a multiple of 7, Player B can ensure that the number of counters decreases by 2 or 7 (by taking 1, 4 and 3 counters after Player A takes 1, 3 and 4, respectively). This way, Player B can always end on a "winning" number.

Furthermore, Player A can never reach numbers which are multiples of 7 or two more than a multiple of 7 (if they start on one of those numbers). This is because going between those numbers requires decreasing the number of counters by 2, 5 or 7, which cannot be done in one move.

Of course, Player B can only carry out this strategy if Player A initially starts on a number which is either a multiple of 7 or 2 more than a multiple of 7, so we need to count how many such numbers there are between 100 and 200 inclusive. There are 14 full blocks of 7 numbers from 100 to 197, with two winning numbers in each block. Additionally, 198 is two more than a multiple of 7, so there is a total 29 starting values of N from which Player B has a winning strategy.

Answer: (a) 332 (b) (i) 443 (ii) 143 (iii) 029