Next Selection Test: 4 hours 30 minutes

Oundle, May 27, 2003

1. Let $p_1, p_2, \ldots, p_n$ be distinct prime numbers greater than 3. Show that $2^{p_1 p_2 \cdots p_n} + 1$ has at least 4^n divisors.

2. Let ABC be a triangle for which there exists an interior point F such that $\angle AFB = \angle BFC = \angle CFA$. Let the lines BF and CF meet the sides AC and AB at D and E respectively. Prove that

$$AB + AC \geq 4DE.$$

3. Let P be a cubic polynomial given by $P(x) = ax^3 + bx^2 + cx + d$, where a, b, c, d are integers and $a \neq 0$. Suppose that $xP(x) = yP(y)$ for infinitely many pairs x, y of integers with $x \neq y$. Prove that the equation $P(x) = 0$ has an integer root.