

# UK IMO Next Selection Test 1

Oundle 2005

1. Let  $a_1, \dots, a_n, \dots$  be positive real numbers such that

$$\frac{a_n + a_{n+2}}{2} < a_{n+1}$$

for every  $n \geq 1$ . Prove that  $a_n < a_{n+1}$  for every  $n \geq 1$ .

2. An acute triangle  $ABC$  is given. Find the locus of points  $M$  in the interior of  $ABC$  such that

$$AB - FG = \frac{MF \cdot AG + MG \cdot BF}{CM}$$

where  $F$  and  $G$  are the feet of the perpendiculars from  $M$  to  $BC$  and  $AC$  respectively.

3. Find all positive integers  $n$  such that  $2^{\phi(n)} - 1$  divides  $n^n$ , where  $\phi$  is the Euler  $\phi$ -function.