Next Selection Test: Exam 1

IMO camp, Oundle School

25-v-2008

Problem 1 Let M and N be vertices of a cube. Assign the number 1 to these vertices and 0 to the other six vertices. A *move* consists of selecting a vertex, and adding 1 to the numbers assigned to each of the three adjacent vertices. Give necessary and sufficient conditions on M and N for there to exist a finite sequence of moves after which all numbers assigned to the eight vertices are equal.

Problem 2 Let X be a point in the interior of triangle ABC. The line AX meets the side BC at A_1. Points B_1 and C_1 are similarly defined. Let R_1, R_2 and R_3 be the respective radii of circles XBC, XCA and XAB, and R be the circumradius of triangle ABC. Prove that

$$\frac{XA_1}{AA_1}R_1 + \frac{XB_1}{BB_1}R_2 + \frac{XC_1}{CC_1}R_3 \geq R.$$

Problem 3 Let $n \geq 2$ be an integer and $a_1, a_2, \ldots, a_n$ be real numbers. Prove that for any non-empty subset S of $\{1, 2, \ldots, n\}$, the following inequality holds.

$$\left(\sum_{i \in S} a_i\right)^2 \leq \sum_{1 \leq i \leq j \leq n} (a_i + \cdots + a_j)^2.$$

Time allowed: 4 hours 30 minutes