Oundle Test 2

28 May 2007

1. For any positive integer n we denote by $f(n)$ the smallest positive integer m such that the sum $1+2+\cdots+m$ is divisible by n. Find all n such that $f(n)=n-1$.

2. The excircle of a triangle ABC touches the side AB and the extensions of the sides BC and CA at points M, N and P, respectively, and the other excircle touches the side AC and the extensions of the sides AB and BC at points S, Q and R, respectively. If X is the intersection point of the lines MN and RS and Y is the intersection point of the lines PN and RQ, prove that the points X, A and Y are collinear.

3. Three schools A, B and C participate in a chess tournament with five students from each school. Let
$$a_1, a_2, \ldots, a_5; b_1, b_2, \ldots, b_5; c_1, c_2, \ldots, c_5$$
be the list of the players from A, B, C respectively. Let P_A, P_B, P_C be the scores of schools A, B, C, respectively, at the end of the tournament. The rules of the tournament are:

   (a) Matches take place sequentially. If a player is defeated then he (or she) is eliminated from the tournament. The first match of the tournament is between the players a_1 and b_1.

   (b) When x_i from school X is defeated by y_j from Y, if there are players left from the third school Z then y_j has another match with a player from Z; if there is no player left in Z but there are players from X then y_j has a match with x_{i+1}. The tournament is over when two schools have been eliminated.

   (c) School X adds 10^{i-1} points to its score when x_i from X wins a match.

   Find the remainder of the number of possible triples (P_A, P_B, P_C) when it is divided by 8.