

NST 2, Eton and Oundle, 2009

1. There are 2008 red cards and 2008 white cards. They are shuffled and dealt to 2008 people seated facing inwards in a circle so that each person has two cards. At every turn of the game, each player simultaneously hands a card to the person on his or her left. If a player possesses at least one red card, then he must pass a red card, but if not he passes a white card. Determine the maximum possible number of turns necessary to reach a state where each player has one card of each colour.
2. Determine the functions $f : \mathbb{R} \rightarrow \mathbb{R}$ which satisfy

$$f(x+y)f(f(x)-y) = xf(x) - yf(y)$$

for all real numbers x and y .

3. Let ABC be a triangle. Its incircle is tangent to AB at E , while the excircle opposite A is tangent to AB at F . Let D be the point on BC for which the incircles of triangles ABD and ACD have equal radii. The lines DE and DB meet the circumcircle of triangle ADF for a second time at X and Y . Show that $XY \parallel AB$ if, and only if, $AB = AC$.