NST 2

31 May 2010

1. The sequence (a_i) is defined by $a_0 = 2$, $a_1 = 1$, and for $n \geq 2$ we let $a_n = a_{n-1} + a_{n-2}$. Show that if p is a prime divisor of $a_{2k} - 2$, then p also divides $a_{2k+1} - 1$.

2. Let I be the incentre of triangle ABC. The incircle touches AB and BC at X and Y respectively. The line XI meets the incircle again at M. Let X' be the point of intersection of AB and CM. The point L on the segment $X'C$ is such that $X'L = CM$.

   Prove that A, L and Y are collinear if, and only if, $|AB| = |AC|$.

3. Let a, b and c be positive real numbers. Suppose that $a^2 + b^2 + c^2 + abc = 4$. Show that $a + b + c \leq 3$.

Each problem is worth 7 points.
Time: 4 hours 30 minutes.