

## NST 2

Oundle, 27 May 2013

1. Let  $P = \{2, 3, 5, 7, 11, \dots\}$  denote the set of all primes less than  $2^{100}$ . Show that

$$\sum_{p \in P} \frac{1}{p} < 8.$$

2. A triangle  $ABC$  is given. Let  $D$ ,  $E$  and  $F$  be points on the lines  $BC$ ,  $CA$  and  $AB$  (respectively) such that  $AF = EF$  and  $BF = DF$ . Prove that the orthocentre of triangle  $ABC$  lies on the circle  $DCE$ .
3. The integer  $n$  is at least 3. Let  $a_1, a_2, \dots, a_n$  be non-negative real numbers which add up to 2. Let  $m$  and  $M$  be the minimum and maximum (respectively) of  $a_1, a_2, \dots, a_n$ . Prove that

$$\sum_{i=1}^n \frac{a_i^4}{1 + a_i^2} \leq \frac{1}{2} + \frac{(M^2 - m^2)^2}{(1 + M^2)(1 + m^2)}.$$