# NST 2

Oundle, May 26 2014

1. Let $\mathbb{N}$ denote the set of positive integers. Find all functions $f : \mathbb{N} \longrightarrow \mathbb{N}$ such that $m^2 + f(n)$ divides $mf(m) + n$ for all positive integers $m$ and $n$.

2. In triangle $ABC$, the centres of the the excircles are $I_a, I_b$ and $I_c$ in the natural notation. The excircle opposite $A$ has contact points $P$ and $Q$ on lines $AB$ and $AC$ respectively. The line $PQ$ intersects the lines $BI_a$ and $CI_a$ at $D$ and $E$ respectively. Let $A_1$ be the intersection of $DC$ and $BE$. The points $B_1$ and $C_1$ are defined analogously. Prove that $AA_1$, $BB_1$ and $CC_1$ are concurrent.

3. In the country of *Mathmania* the cities are connected by direct two-way internal flights, and this problem concerns only these cities and these flights. It is possible to go from any city to any city by a sequence of flights. The *distance* between two cities is the least number of flights needed to go from one to the other. Each of these cities has at most 100 cities at distance exactly three from it. Prove that there is no city such that more than 2550 other cities have distance exactly four from it.