The 17$^{\text{th}}$ Romanian Master of Mathematics Competition

Day 1: The 25$^{\text{th}}$ of February, 2026, Bucharest

Language: English

Problem 1. Let n be a positive integer. Alice draws a unit area triangle on the board. Then she draws additional triangles by performing n moves in a row. On each move, she chooses a drawn triangle Δ with no marked points in its interior, marks a point P in its interior, and draws three smaller triangles by joining P to each vertex of Δ with a segment.

Once these n moves have been performed, Bob chooses three distinct drawn triangles Δ_1, Δ_2, and Δ_3 which contain no marked points in their interiors, such that Δ_2 shares one side with Δ_1 and another with Δ_3. In terms of n, determine the largest possible constant c such that Bob can guarantee that the sum of the areas of Δ_1, Δ_2, and Δ_3 is at least c, regardless of Alice's choices.

Problem 2. Let $p \geq 11$ be a prime. Suppose that, if a and b are integers such that $1 \leq a < b \leq p - 3$, then $b! - a!$ is not divisible by p. Prove that $p - 5$ is divisible by 8.

Problem 3. Let $\mathcal{S}$ be a finite subset of $\mathbb{R}^3$. Prove that there exist three polynomials $P(x, y, z)$, $Q(x, y, z)$ and $R(x, y, z)$ with real coefficients, such that a triple of real numbers (a, b, c) is in $\mathcal{S}$ if and only if the system of equations

$$P(x, y, z) = a,$$

$$Q(x, y, z) = b,$$

$$R(x, y, z) = c,$$

does **not** have a solution in real numbers x, y, and z.

Each problem is worth 7 marks.
Time allowed: $4\frac{1}{2}$ hours.