

The 17th Romanian Master of Mathematics Competition

Day 2: The 26th of February, 2026, Bucharest

Language: English

Problem 4. For any positive integer m , let $\varphi(m)$ be the number of positive integers less than or equal to m and coprime to m . Define $\varphi_0(m) = m$ and, for each positive integer k , $\varphi_k(m) = \varphi(\varphi_{k-1}(m))$. For any integer $n \geq 3$, prove that

$$\varphi_0(2^n - 3) \cdot \varphi_1(2^n - 3) \cdot \varphi_2(2^n - 3) \cdot \dots \cdot \varphi_n(2^n - 3)$$

has at most n distinct prime divisors.

Problem 5. Let ABC be a triangle with $AB < AC$, let O be its circumcentre and let $XYZT$ be a parallelogram inside triangle ABC such that

$$\angle AXB = \angle AZC, \quad \angle AZB = \angle AXC,$$

$$\angle AYB = \angle ATC, \quad \angle ATB = \angle AYC.$$

Prove that the diagonals XZ and YT of the parallelogram intersect on the circumcircle of BOC .

Problem 6. Let $k > 1$ be an integer, and let S denote the set of all $(k+1)$ -tuples of integers $X = (x_1, \dots, x_{k+1})$ such that $1 \leq x_1 < \dots < x_{k+1} \leq k^2+1$. If σ is a permutation of the numbers $1, 2, \dots, k^2+1$, say that an element X of S is σ -nice if the sequence $\sigma(x_1), \sigma(x_2), \dots, \sigma(x_{k+1})$ is monotone. Prove that

$$\min_{1 \leq i \leq k} \left\lfloor \frac{x_i}{i} \right\rfloor + \min_{2 \leq i \leq k+1} \left\lfloor \frac{k^2 + 2 - x_i}{k + 2 - i} \right\rfloor \geq k + 1$$

if and only if there exists a permutation σ such that X is the unique σ -nice tuple in S .

Each problem is worth 7 marks.

Time allowed: $4\frac{1}{2}$ hours.