THE MATHEMATICAL ASSOCIATION

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NATIONAL COMMITTEE FOR MATHEMATICAL CONTESTS.

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National Committee for Mathematical Contests

Reading Selection Test, 1987.

1 A, B, C are angles of an acute-angled triangle. Prove

$$(\tan A + \tan B + \tan C)^2 \geqslant (\sec A+1)^2 + (\sec B+1)^2 + (\sec C+1)^2 .$$

2 Prove that the integer next greater than $(3+\sqrt{5})^n$ is divisible by 2^n.

3 Q is a convex quadrilateral whose 4 vertices are on the circumference of a circle whose centre is O.
The distance of any side of Q from O is half the length of the opposite side.
Prove that the diagonals of Q intersect orthogonally.

4 Simplify $(x^2 + 3x + 1)^2 - 5x(x + 1)^2$. Hence, or otherwise, express $5^{15} - 1$ as the product of 3 integers, one greater than 100 and the other 2 each greater than 10,000.

5 Each root of the polynomial equation

$$P(x) \equiv x^{100} - 600x^{99} + a_{98}x^{98} + a_{97}x^{97} + \dots\dots\dots + a_1 x + a_0 = 0$$

is real. $P(7)$ is greater than 1.

Prove that some root of $P(x) = 0$ is greater than 7.