

NATIONAL COMMITTEE FOR MATHEMATICAL CONTESTS

Second International Selection Test

Cambridge, 16th April 1989

Time allowed : 3½ hours

*Please write on one side of the paper only, use separate sheets for each question, and put your name on each sheet.*

1. Find the average of the quantity  
$$(a_1 - a_2)^2 + (a_2 - a_3)^2 + \dots + (a_{n-1} - a_n)^2$$
taken over all permutations  $(a_1, a_2, \dots, a_n)$  of  $(1, 2, \dots, n)$ .
  
2. A positive integer is called a double number if its decimal representation consists of a block of digits, not commencing with 0, followed immediately by an identical block. So, for instance, 19891989 is a double number, but 89089 (= 089089) is not. Show that there are infinitely many double numbers which are perfect squares, and give an example of one.
  
3. Let  $A$  be a set of positive integers such that for any two distinct elements  $x, y$  of  $A$ ,  $|x - y| \geq xy/25$ . Prove that  $A$  contains at most nine elements. Give an example of such a set of nine elements.
  
4. The triangle  $ABC$  is acute-angled.  $L$  is any line in the plane of the triangle and  $u, v, w$  are the lengths of the perpendiculars from  $A, B, C$  respectively to  $L$ . Prove that
$$u^2 \tan A + v^2 \tan B + w^2 \tan C \geq 2\Delta$$
where  $\Delta$  is the area of the triangle, and determine the lines for which equality holds.