UK Mathematical Olympiad for Girls

EGMO | 2012
European Girls' Mathematical Olympiad

23 June 2011

Instructions

Supported by
MILLFIELD

- *Time allowed: 3 hours.*
- *Full written solutions – not just answers – are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.*
- *One complete solution will gain more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all the problems.*
- *Each question carries 10 marks. However, earlier questions tend to be easier. In general you are advised to concentrate on these problems first.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Start each question on a fresh sheet of paper. Write on one side of the paper only. On each sheet of working write the number of the question in the top left hand corner and your name, initials and school in the top right hand corner.*
- *Complete the cover sheet provided and attach it to the front of your script, followed by your solutions in question number order.*
- *Staple all the pages neatly together in the top left hand corner.*
- *To accommodate candidates sitting in other timezones, please do not discuss the paper on the internet until 8am BST on Friday 24 June.*

Do not turn over until **told to do so**.

A partnership of

United Kingdom Mathematics Trust

Murray Edwards College
founded as New Hall

UK Mathematical Olympiad for Girls

23 June 2011

1. Three circles MNP, NLP, LMP have a common point P. A point A is chosen on circle MNP (other than M, N or P). AN meets circle NLP at B and AM meets circle LMP at C. Prove that BC passes through L.

   Many diagrams are possible. You need only solve this problem for one of the possible configurations.

2. The number 12 may be factored into three positive integers in exactly eighteen ways, these factorizations include $1 \times 3 \times 4$, $2 \times 2 \times 3$ and $2 \times 3 \times 2$. Let N be the number of seconds in a week. In how many ways can N be factored into three positive integers?

   A numerical answer is not sufficient. The calculation should be explained and justified.

3. Consider a convex quadrilateral and its two diagonals. These form four triangles.

   (a) Suppose that the sum of the areas of a pair of opposite triangles is half the area of the quadrilateral. Prove that at least one of the two diagonals divides the quadrilateral into two parts of equal area.

   (b) Suppose that at least one of the two diagonals divides the quadrilateral into two parts of equal area. Prove that the sum of the areas of a pair of opposite triangles is half the area of the quadrilateral.

4. Find a cubic polynomial $f(X)$ with integer coefficients such that whenever a, b, c are real numbers such that $a+b+c = 2$ and $a^2+b^2+c^2 = 2$, we have $f(a) = f(b) = f(c)$.

5. Let a be an even integer. Show that there are infinitely many integers b with the property that there is a unique prime number of the form $u^2 + au + b$ with u an integer.

   Note that an integer p is "prime" when it is positive, and it is divisible by exactly two positive integers. Therefore 1 is not prime, nor is -7.

Time allowed: 3 hours